Modelling the afforestation and underlying uncertainties

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Abstract

A dynamic model of carbon budget of oak forest ecosystem which takes into account forest stand age is developed. A numerical experiment is designed to simulate an afforestation process and a Monte-Carlo simulation was performed to determine how the parameter uncertainties influence the result.

1 Introduction

Afforestation (planting of forests on lands were forests did not grow for last 50 years) is a "natural" way of trapping the atmospheric carbon dioxide in long-living phytomass, detritus and humus. When planning an afforestation it is important to estimate amount of carbon that can be accumulated in the ecosystem during some period, uncertainty of the estimation and risk of not achieving the desired result. The point is that the forest should grow in changing environment, besides the estimator is uncertain itself.

It is proposed to use a dynamic mathematical model of carbon budget of oak forest ecosystem with growth functions [1] and regression expressions [2] for taking into account forest stand age. For description of phenology a function of monthly mean temperature is developed. A simple mathematical model for estimation of available water in ecosystem is elaborated for involving the effect of accumulation of frozen water in winter and melting in spring. The uncertainties of the model parameters (including temperature, precipitations and atmospheric carbon dioxide) are modeled with random generators of certain probability distributions.

The study is illustrative since the information about the model parameters is incomplete and thus uncertainty classes 10% and 20% of relative standard deviation as well as assumptions on probability distribution types (normal or uniform) were done.

2 Description of the model and experiment

2.1 Description of the model.

In the mathematical model of carbon budget of oak forest the following carbon pools are considered: phytomass (leaves are distinguished using regression expression), litter (five reservoirs: foliage, stems, branches, coarse roots, and fine roots), and soil organic matter; and also the following carbon flows are considered: atmosphere-phytomass, phytomass-litter (litter sorting into five types using regression expressions), litter-atmosphere, litter-soil, soil-atmosphere, phytomass-boundary of the ecosystem (harvested phytomass).

Mathematical model of carbon budget is presented in the form of system of ordinary differential equations of first order

$$\begin{aligned}
\frac{dX_{ph}}{dt} &= v_{ap} - (v_{plf} + v_{pls} + v_{plb} + v_{plcr} + v_{plfr} + v_{ph}) \,, \\
\frac{dX_{lf}}{dt} &= v_{plf} + v_{hlf} - (v_{lfa} + v_{lfs}) \,, \\
\frac{dX_{ls}}{dt} &= v_{pls} + v_{hls} - (v_{lsa} + v_{lss}) \,, \\
\frac{dX_{lb}}{dt} &= v_{plb} + v_{hlb} - (v_{lba} + v_{lbs}) \,, \\
\frac{dX_{lcr}}{dt} &= v_{plcr} + v_{hlcr} - (v_{lcra} + v_{lcrs}) \,, \\
\frac{dX_{lfr}}{dt} &= v_{plfr} + v_{hlfr} - (v_{lfra} + v_{lfrs}) \,, \\
\frac{dX_{s}}{dt} &= v_{lfs} + v_{lss} + v_{lbs} + v_{lcrs} + v_{lfrs} - (v_{sa} + v_{saq}) \,,
\end{aligned}$$

where letter X with subscripts denotes carbon pools (ph – phytomass, lf – foliage litter, ls – stem and branch (diameter greater than 10cm) litter, lb – branch litter (diameter less than 10cm), lcr – coarse root litter, lfr – fine root litter), and letter v with subscripts denotes carbon flows between corresponding reservoirs, e.g., ap – atmosphere-phytomass, plf – phytomass-foliage litter, ph – phytomass-harvested phytomass, saq – soil-aquatic system.

Photosynthesis intensity (flow v_{ap}) is presented with a complex function:

$$v_{ag} = \alpha_{ap} * F_l * \min\{F_T, F_c, F_w\}$$

where α_{ap} – calibration coefficient, F_l – function of mass of leaves which in its turn is a function of forest stand age (A), F_T – dependence on the monthly air temperature (T), F_c – dependence on monthly concentration of atmospheric CO_2 (C) and F_w – dependence on monthly amount of available water (w). Let's consider the main functions.

Mass of leaves (denoted as f) is defined with regression equation (see expression (1), described below), but time of appearance of leaves in oak forests is controlled with air temperature (T_{lg}) using expression:

$$F_1 = \frac{1}{1 + \exp(0.9 * (-T + T_{\mathrm{lg}}))} * \frac{R_f * X_{ph}}{R_{tot}}.$$

Functions F_T, F_c, F_w are defined in [3]. Optimal temperature of photosynthesis is chosen equal to normal temperature of July in corresponding vegetation belt, which for oak forests is $18,4^o$C.

Flow phytomass-foliage litter (v_{plf}). Time of fall of the leaves is controlled with air temperature which is decreasing (T_{lfb} – temperature of mass fall, T_{lfe} – temperature when the fall of the leaves ends; see Table 14), but the intensity is controlled with mass of the leaves:

$$v_{plf} = \begin{cases} 15\left(\frac{1}{1+\exp(1.2(T-T_{lfe}))} - \frac{1}{1+\exp(1.2(T-T_{lfb}))}\right)\frac{R_f X_{ph}}{R_{tot}}, & if\ \frac{dT}{dt} < 0 \\ 0, & otherwise. \end{cases}$$

Monthly mean air temperatures of major phenology changes in the ecosystems of oak forests is compiled using observations in Natural Reserve "Roztochchya". The temperature of appearance of leaves (T_{lg}) is 12.5^{o}C, temperature of mass fall of the leaves (T_{lfb}) is 9.0^{o}C and temperature of end of fall of the leaves (T_{lfe}) is 5.7^{o}C.

Phytomass is divided into fractions (stems and branches of diameter greater than 10cm – denoted as s, branches of diameter less than 10cm – denoted as b, coarse roots – cr, and fine roots – fr) using regression expression [4]

$$R_i = a_0^i * A^{a_1^i},$$

where a_0 and a_1 are regression coefficients values which are listed in [2]. Also the following relations between the regression expressions are used: $R_{br} = R_{kr} - R_f$, $R_{fr} = R_f$, $R_{cr} = R_{bl} - R_{fr}$, $R_{st} = R_{ab} - R_{kr}$.

Carbon flows from phytomass to corresponding litter reservoirs are defined with the following expression

$$v_{pli} = \alpha_{pli} * \left(dM * R_i * \frac{X_{ph}}{GS * R_{tot}} + \frac{X_{ph*R_i}}{Turn_i}\right), \quad i = \{s, b, cr, fr\},$$

where α_i is calibration coefficient, dM is natural mortality of forest stand defined in [1]. $Turn$ – turnover time for tree parts listed in [4]. For usability we assume that for stem $Turn = \infty$.

Litter mineralization is described with the following expression:

$$V_{lia} = kle * F_{phi} * F_{Tl} * F_{Pl} * X_i, \quad i = \{f, s, b, cr, fr\},$$

where kle is a calibration coefficient, F_{phi} is a function of phytomass amount [5]

$$F_{phi} = k_i + 0.5 * k_i \exp\left(-\frac{9.21 * X_{ph}}{0.065 * GS}\right),$$

$k = 0,045 \div 0,42$ [4]. F_{Tl} is a function of temperature, it is defined in [6], the parameter Q_{10} equals $2,25$ [7]. F_{Pl} is a function of water amount

$$F_{p_l} = 1 - \exp\left(-0.017 * w\right).$$

Mineralization of soil organic matter (V_{sa}) is determined with similar expression but different parameters are used, particularly, $Q_{10} = 1,84$ [7], and $f_{ph} = 1$.

Litter humification: $V_{lis} = p * V_{lia}$, $i = \{f, s, b, cr, fr\}$ ($p = 0,19$). One assumes that branches, roots and stubs of harvested trees are left in the forest: $V_{hli} = Rv_i * Harv$, $i = \{f, s, b, cr, fr\}$; $V_{hls} = 0,3 * Harv$, where $Harv$ is amount of harvested stemwood. Flow $vsqa$ is introduced for taking into account increasing of soil run-off when forests are harvested. In current version of the model the flow is set constant 0,0004 kgC/(m^{2}year).

2.2 Submodel of available water in ecosystem

Let describe dynamics of snow with the following equation

$$\frac{dm_s}{dt} = -(v_a + v_p + v_w) * (1 - \exp(-2 * m_s)) * \gamma,$$

where v_a – snow, melted because of heat exchange between snow and air (equation of ideal gas energy is used)

$$v_a = \begin{cases} \frac{3}{2} * \frac{m_{air}}{\mu} * R * \frac{T}{\lambda} & if \quad T > 0 \quad and \quad m_s > 0 \\ 0 & if \quad T < 0 \quad or \quad m_s = 0 \end{cases},$$

in the equation m_{air} denotes mass of air near the ground, kg/m^2; μ – molar mass of air (0,029kg/mol), T – air temperature, K; λ – specific heat of ice melting ($3,34\ 10^5$J/kg).

v_p – snow melted because of heat exchange between snow and rain water, and also kinetic energy of rain:

$$v_p = \begin{cases} \frac{C_w * W_i * (T+5) + 0.5 * W_i * v^2}{\lambda} & if \quad T > 0 \quad and \quad m_s > 0 \\ 0 & if \quad T < 0 \quad or \quad m_s = 0, \end{cases}$$

where C_w – specific heat of water ($4,3\ 10^3$J/(kg K)), W_i – precipitations, kg/m^2; v – mean vertical speed of rain droplets near the earth surface (6,5m/s [8]);

v_{SR} – snow melted because of snow heating with solar radiation

$$v_{SR} = \begin{cases} \frac{SR*(1-\alpha)}{\lambda} & if \quad T > 0 \quad and \quad m_s > 0 \\ 0 & if \quad T < 0 \quad or \quad m_s = 0, \end{cases}$$

where SR – solar radiation, W/(m^{2}year), α – forest albedo (0,15).

v_{wt} – snow, weathered and blown by the wind, $v_{wt} = 0.1 * m_s$.

There is also a dimension factor, $\gamma = 1\ s^{-1}$.

Water amount which is available in the ecosystem and influence the intensity of carbon cycle processes, W_a, is defined with the following expression

$$W_a = \begin{cases} W_i + v_a + v_p + v_{SR} - v_{wt} & if \quad T > 0 \quad and \quad m_s > 0 \\ 0 & if \quad T < 0 \\ W_i & if \quad T > 0 \quad and \quad m_s = 0. \end{cases}$$

2.3 Calibration and testing of the model.

For calibration and testing of oak forest model measurement data from four test plots of oak forests of different age (33, 54, 75 and 106 years) [9] are used. Root-mean-square error of phytomass modeling is 16% and phytomass net increment – 24% (Table 1).

* Values reduced to forest stand stocking 0,79.

2.4 Modeling of the parameter uncertainties.

The model includes a number of parameters. Most of the parameters are taken from literature and derived from field experiments. Of course the parameters are uncertain. Usually, the parameter uncertainties are poorly reported. For the parameters we apply

	Age, years	33	54	75	106	Root-mean-square error, %
Phytomass, kgC/m^2	Measured	5,40	8,70*	11,56*	13,67*	16
	Modeled	5,40	10,05	12,99	15,58	
	Difference	0,00	1,35	1,43	1,91	
Net increment, kgC/(m^{2}year)	Measured	0,36	0,17	0,16	0,11	24
	Modeled	0,34	0,16	0,12	0,05	
	Difference	0,02	0,01	0,04	0,06	

Table 1: Comparison of measured and modeled phytomass and net increment of oak forest.

uncertainty classes: 10 or 20% of standard deviation. If we do not have any information about probability distribution a uniform distribution is assumed. The parameter relative uncertainties and probability distributions are as folows (U – uniform distribution and N – normal distribution; e.g. $10U$ means 10% of uniform distribution): $T - 10N$, $T_{opt} - 10U$, $\alpha - 10U$, $C - 10N$, $\beta - 10U$, $W - 10N$, $k_w - 20U$, $GS - 20N$, $dM - 20N$, $R_i - 20N$, $k_i - 20U$, $Q_{10} - 20U$, $p - 20U$. A Monte-Carlo simulation was performed to determine how the parameter uncertainties influence the result. For that the uncertain parameters were modelled with generators of random values with uniform or normal probability distributions. Besides the initial conditions were modelled with 20% uncertainty of uniform probability. The system of differential equations was solved 650 times.

2.5 Numerical experiment.

Oak forest (III forest site capacity) is planted on the place of cropland. As initial data about carbon stored in modelled ecosystem components (phytomass, litter and soil) the results, produced by carbon balance model of cropland [3] was used. Monthly mean temperature, precipitation and solar radiation averaged over long period for Lviv Region, and monthly mean atmospheric carbon dioxide were used for modeling. Planted forest is monitored for 20 years.

3 Results and Discussion

Amount of phytomass increases from 0,6 kg C/m^2 to 2,7 kg C/m^2 in 20 years (Fig. 1), relative standard deviation changes from 20% at the beginning of experiment to 37% at the end. Carbon stock of litter (includes leaves, branches, stems, coarse roots and fine roots) increases gradually from zero having a maximum at 0,5 kg C/m^2 in 6 years, and after that it decreases to 0,2 kg C/m^2 during next 15 years (Fig. 2). Such litter dynamics can be interpreted by age-specific changes of forest. In this case relative standard deviation increases from 20% at the beginning of experiment to 83% at the end. Carbon stock of soil organic matter decreases slightly (during first two years) because of small litter supply (humus mineralizes faster than develops); in the next years it increases from 1,3 to 1,5 kg C/m^2 (Fig. 3). But relative standard deviation decreases from initial 20% to 15%.

Total carbon amount, stored in the system, increases from 1,9 kg C/m^2 to 4,4 kg C/m^2 during next 20 years (Fig. 4). Relative standard deviation increases from 9% to 21%. Histogram of the total carbon amount at the end of modeling is shown in Fig. 5 and box plot – in Fig. 6. The boxes represent lower quartile, median, and upper quartile values. The solid lines in both sides of the boxes comprise values in the range ±1,5 times of inter-quartile distance. Other values ("outliers") are plotted with "+". Outliers represent a risk of underestimation or overestimation of accumulated carbon in the planted forest. In this case the risk of underestimation is greater which is positive.

Contribution of variation of such parameters as temperature, precipitations and atmospheric carbon dioxide to total uncertainty is substantial comparing to the uncertainty caused by the other model parameters. For example, relative standard deviation at 10 modelling year for phytomass increases two times, the uncertainties of soil compartment and total accumulated carbon increase almost 1,4 times, whilst the uncertainty of litter compartment stays almost on the same level (Table 2). Number of outliers also increases.

	Phytomass, kg C/m^2		Litter, kg C/m^2		Soil, kg C/m^2		Total, kg C/m^2	
	Value	std	Value	std	Value	std	Value	std
Without climatic parameters and CO_2	2,1	0,5 22%	0,4	0,2 43%	1,4	0,2 12%	4,0	0,7 16%
Including climatic parameters and CO_2	2,3	1,0 44%	0,4	0,2 44%	1,4	0,3 16%	4,1	0,9 22%

Table 2: Accumulated carbon and its standard deviation after 10 years of Monte-Carlo simulation without climatic parameters and atmospheric CO_2 and including climatic parameters and atmospheric CO_2

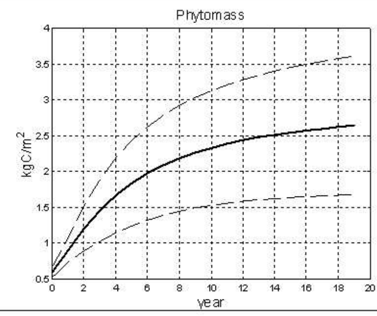

Fig.1. Dynamics of phytomass carbon and its standard deviation.

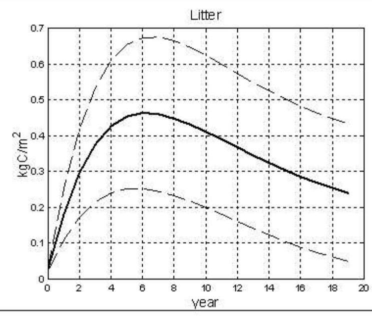

Fig.2. Dynamics of litter carbon and its standard deviation.

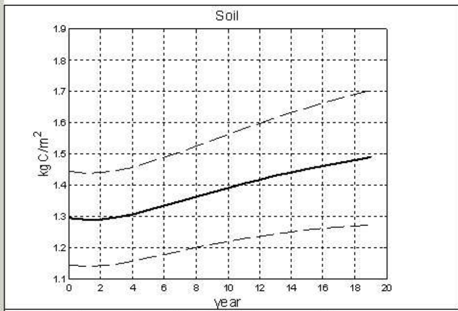

Fig.3. Dynamics of soil carbon and its standard deviation

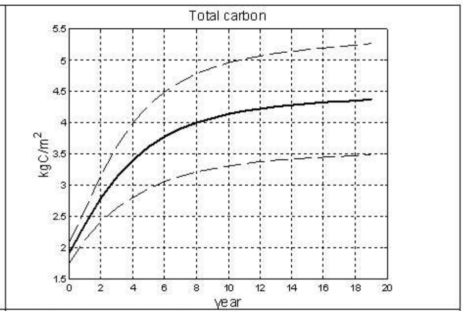

Fig.4. Dynamics of total ecosystem carbon and its standard deviation

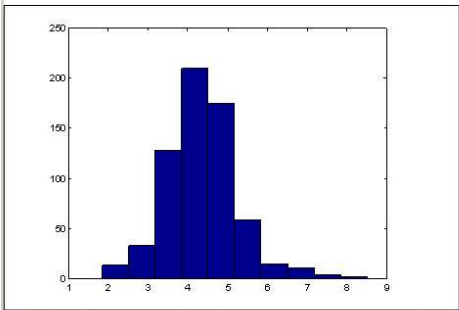

Fig.5. Histogram of total ecosystem carbon at the end of modelling

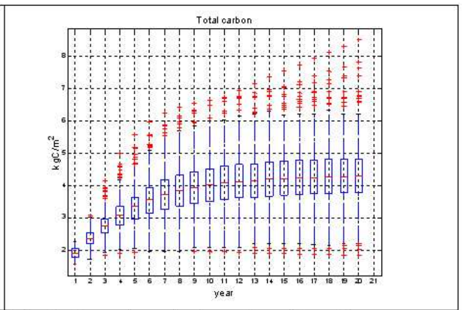

Fig.6. Box-plot of dynamics of total ecosystem carbon; the boxes represent lower quartile, median, and upper quartile values; the solid lines in both sides of the boxes comprise values in the range ±1,5 times of inter-quartile distance; outliers are plotted with "+".

3.1 Conclusions

A dynamic mathematical model of carbon budget of oak forest accounting for forest stand age and water in available form is proposed for study afforestation. The model suits well the calibration data – 16% RMSE in phytomass accumulation and 24% RMSE in net increment for 73 years of modeling. Developed mathematical model of the carbon budget of oak forest allows prognosing the carbon budget of ecosystems at changing climate and atmospheric carbon dioxide. Introduction of functions of forest stand age allows reproducing the forest age dynamics of major carbon stocks and flows in ecosystem.

A Monte-Carlo simulation of afforestation of degraded land was performed to study influence of model parameter uncertainties (including variation of temperature, precipitations and atmospheric carbon dioxide) on the result uncertainty. The forest growth was monitored for 20 years after planting.

Uncertainty of litter accumulation is the largest: relative standard deviation increases from 20% at the beginning of experiment to 83% at the end. But portion of litter is small in the overall phytomass and so it does not influence the total uncertainty substantially. Relative standard deviation of phytomass carbon changes from 20% at the beginning of experiment to 37% at the end. And relative standard deviation of soil carbon is the smallest and even decreases from initial 20% to 15%. Relative standard deviation of total carbon accumulated in the ecosystem increases from initial 9% to 21% in 20 years. From the point of view of verification time concept the signal is detectable if one considers 1-sigma confidence interval, but in case of 3-sigma the signal becomes non-detectable[2] . Number of outliers is quite high. Outliers represent a risk of underestimation or overestimation of accumulated carbon in the planted forest. In this case the risk of underestimation is greater which is positive.

Variation of temperature, precipitations and atmospheric carbon dioxide influence the total uncertainty substantially. In 10 modelling years relative standard deviation of phytomass increases two times, uncertainty of soil compartment and total accumulated carbon increase almost 1,4 times, whilst the uncertainty of litter compartment stays almost on the same level. Number of outliers also increases. Thus the climate variability must be taken into account when one prognoses the gain of an afforestation project.

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[2]We consider prognostic modelling