Performance of the carbon market when accounting for uncertainties in GHG inventories

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Abstract

The aim of the paper is to examine with a real-world data the already developed framework of emission trading under big uncertainty in GHG inventories. The trade is performed with *the effective* emission permits. They differ from standard permits since they reflect a participant inventory quality. The starting point is undershooting the Kyoto targets with a specified risk due to uncertainty. As this introduces extremely high total abatement costs, we examine also appropriate adjustment of the Kyoto obligation. Parties become penalized for differing from arbitrarily chosen level of uncertainty. The adjustment proves to be very practical solution as simulation results show.

1 Introduction

The idea of permit trading has been established in order to contribute to achieving environmental goals. In the context of limited funds that can be spent on environment protection it promotes cost effectiveness. Implementation of the Kyoto treaty opens new perspective for carbon trading on international scale.

The Kyoto framework relies on the system of national emission inventories. Uncertainties underlying such large-scale inventories are unavoidable especially in the situation when greenhouse gas emissions are generally not directly measurable. Estimates made so far show the range of 10% for Austria to 19% for United Kingdom to 21% for Norway [8]. GHG emission permits are far from being a robust commodity to trade. The question arises how the uncertainties will influence financial liabilities among parties.

The idea of permit trading rests on the heterogeneity of emission reduction costs among the market participants, including differences in technology, experience as well as availability of natural resources etc. We aim to explicitly include also diversified inventory performance. In the long run this would stimulate further improvements in the field. Also in [8] consideration of the system that "allow a market-based encouragement to reduce emission uncertainty" has been highly recommended.

In [5], see also [6], the following scheme was proposed. First, the concept of undershooting was used - a part of uncertainty level has to be added to emission level when proving compliance with the Kyoto target. More precisely, in order to establish to which extent the uncertainty level is to be considered, the authors introduced a risk that real, unknown emissions actually exceed reported level due to inventory uncertainties. Secondly, it is assumed that the uncertainty of the purchased emissions contributes to the overall buyer's uncertainty. What follows is that the value of emissions becomes differentiated on the basis of uncertainties underlying emission inventories. These differences are compensated by so called *the effective* emission permits. Because undershooting introduces high additional abatement costs, it is proposed to adjust the commitment obligations for each party, taking into account the proportion between its own uncertainty and an arbitrarily chosen level of uncertainty.

The aim of this paper is to employ the above-mentioned ideas into a market optimization problem, i.e. to simulate trading with effective permits within the undershooting framework and then with appropriately adjusted commitment obligation.

The Kyoto protocol, as it is formulated right now, requires just reporting of uncertainties underlying inventories. It does not mention any form of undershooting. Therefore we develop here a kind of a blueprint.

The paper is constucted as follows. Section 2 formulates the standard task of market opimization that will be of use in the sequel. Section 3 scrutinizes the most important ideas from [5] and puts them into emission trading perspective. Section 4 presents the results of numerical simulation and Section 5 concludes.

2 The benchmark case

The following notation will be used:

$i = (1, 2..., N)$ – a party of the Kyoto Protocol;

x_i – emission level;

$c_i(x_i)$ – cost of keeping (reducing) emission at level x_i;

δ_i – fraction of the party base year emission that is to be reduced according to the Kyoto obligation;

x_i^0 – base year emission.

The task is to meet the target of the Kyoto protocol and not to allow costs to become higher than it is necessary [1, 10]:

$$\begin{array}{c} \min_{x_i} \sum_i c_i(x_i) \\ \text{s.t. } \sum_i (x_i - (1-\delta_i)x_i^0) = 0 \end{array} \tag{1}$$

The border condition takes the form of equation, as we assume parties never overcomply. Constructing the Lagrangian we obtain the condition for the static market equilibrium:

$$\lambda = -\frac{\partial c_i(x_i)}{\partial x_i}$$

where λ is the Lagrange multiplier being iterpreted as the market-clearing price.

3 Uncertain emission inventory

Effective emission permits. The relationship between the reported emission level x_i and the effective emission permits l_i is straightforward [5]:

$$l_i = [1 - (1 - 2\alpha)R_i]x_i \tag{2}$$

where

α – a risk that the party is actually non-compliant. The parameter α is to be set beforehand, common for all market participants;

R_i – relative uncertainty underlying inventories [1].

The formula directly reflects the following rule - higher the uncertainty less units of effective emission permits a party is allocated with. Since effective permit will be the standard permit used in our setting, also cost of emission abatement has to be expressed in terms of effective permit units:

$$c_i(x_i) = c_i(\frac{l_i}{1 - (1 - 2\alpha)R_i}) \tag{3}$$

This shifts the argument of the abatement cost function according to party's uncertainty level R_i and assumed risk level α. Market decisions will be made on the basis of those shifted cost functions.

Kyoto targets when accounting for uncertainty in both the commitment and base year - Undershooting concept. The idea behind is that the emission level in the commitment year is not to exceed the Kyoto agreed obligation decreased by additional uncertainty belt Δ_i^{bc} defined in the following way:

$$\Delta_i^{bc} = \Delta_i^c + (1 - \delta_i)\Delta_i^b \tag{4}$$

where Δ_i^c, Δ_i^b stand for the half absolute uncertainty intervals in the commitment and base year, respectively [2]. Also a risk α has been introduced to allow partial treatment of uncertainty, however, according to established rules. Described rule yields the condition:

$$x_i \leq (1 - \delta_i)x_i^0 - (1 - 2\alpha)\Delta_i^{bc} \tag{5}$$

As a result, the standard reduction factor δ_i is replaced with the factor δ_i^{Up} reflecting the undershooting concept in the context of effective permits:

$$\delta_i^{Up} = \delta_i + 2(1 - 2\alpha)R_i \tag{6}$$

For detailed derivation of the above formula see [5].

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[1] Actually, differentiated levels of uncertainty for the base year and for the commitment year have been considered in [5]. However, in our setting we do not consider the problem of uncertainty reduction. This would require to analyse also costs of such actions (compare [2, 7]).

[2] As already mentioned, in this paper we do not consider diversified uncertainty levels between the commitment and base year. This was made just here in order to bring the message across.

Since our scope is to trade with effective permits under the undershooting rule, the standard commitment condition as formulated in (1) has now to be in line with the following condition:

$$l_i \leq [1 - \delta_i - 2(1-2\alpha)R_i]x_i^0[1-(1-2\alpha)R_i] \tag{7}$$

It differs from the standard solution since the Kyoto original emission endowment is decreased according to the inventory uncertainty R_i and considered risk level α. The last two terms on the right hand side of inequality (7) correspond to the effective permits in the base year.

The cost-effective attainment of the protocol commitments in the context of effective permits can be then characterized by the following model:

$$\begin{array}{c} \min_{l_i} \sum_i c_i(\frac{l_i}{1-(1-2\alpha)R_i}) \\ \text{s.t. } \sum_i (l_i - [1-\delta_i - 2(1-2\alpha)R_i]x_i^0[1-(1-2\alpha)R_i]) = 0 \end{array} \tag{8}$$

Constructing the Lagrangian yields the condition:

$$\lambda = -\frac{\partial c_i(\frac{l_i}{1-(1-2\alpha)R_i})}{\partial l_i} \tag{9}$$

Adjustment of the Kyoto commitment condition. Since undershooting decreases Kyoto emission endowments it will most likely result in considerable increase of abatement costs. Bearing this in mind, Nahorski *et al.* [5] suggested the following solution. The idea is to compare uncertainty level with a reference one, which satisfies the original Kyoto target and also has a chosen uncertainty magnitude R^M. Hence the original Kyoto reduction target δ_i is substituted with the following factor δ_i^{Ap}:

$$\delta_i^{Ap} = \delta_i + (1-2\alpha)(2R_i - R^M) \tag{10}$$

This way parties become penalized just for differing from arbitrarily chosen level of uncertainty R^M.

Few propositions for defining R^M has been given. We will stick to the simplest one, under which R^M reflects average uncertainty among the market participants (more precisely, $R^M = \frac{1}{N}\sum_i \frac{\Delta_i^{bc}}{x_i^0}$).

The adjustment turns the border condition in our optimization model into the following shape (again, for details check [5]):

$$\begin{array}{c} \min_{l_i} \sum_i c_i(\frac{l_i}{1-(1-2\alpha)R_i}) \\ \text{s.t. } \sum_i (l_i - [1-\delta_i + (1-2\alpha)(R^M - 2R_i)]x_i^0[1-(1-2\alpha)R_i]) = 0 \end{array} \tag{11}$$

4 Simulation results

Below we present results of market optimization as formulated in (8) and (11). Data for regional abatement cost functions come from [3][3]. The Kyoto protocol participants are aggregated into five groups: United States, OECD Europe,

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[3] Kind provision of data from Odd Godal is gratefully acknowledged.

Japan, Canada/Australia/New Zeland and finally Eastern Europe/Former Soviet Union. Data on uncertainty level were derived from [2, 8] and partly assumed (for Japan). The results here and particularly in the sequel should be regarded as illustrative and not the ultimate solution due to partly estimated data. Table 1 depicts the situation before any exchange of permits has been made.

	Base year emissions	Kyoto target	Inventory uncertainty	Total costs	Marginal costs
Units	MtC/year	%	%	MUS$	$/tC
US	1345	7,0	13	89 343	-313,7
OECDE	934	7,9	10	28 652	-322,7
Japan	274	6,0	15	21 077	-453,8
CANZ	217	0,7	20	10 477	-216,5
EEFSU	1337	1,7	30	0	0,0
Total	4107			149 549	

Table 1: Base year emissions, committed changes in emissions, inventory uncertainty, total and marginal costs of compliance without trade

One can immediately spot from Table 1 a disappropriate gap between the Kyoto targets and the magnitude of inventory uncertainties. Although some objections can be formulated towards the exact uncertainty levels accepted here, this situation generally follows earlier observations, e.g. [6, 8], revealing potential troubles with the Kyoto protocol compliance.

Trading with effective permits under undershooting. Table 2 shows the results of trading with effective permits according to the idea of undershooting for few parameters α. It starts with $\alpha = 0,5$, which means neglecting uncertainty. Obviosly, effective permits and reported emission are equal and we obtain standard solution with market shadow price 142,5 \$/tC and total abatement costs for all parties considerably diminished from the situation of no trade. EEFSU is the only net seller of permits.

Setting $\alpha = 0,3$ we accept a risk of 30% that a party actuall emission is above the Kyoto target. This is reflected in diversified level of effective permits and reported emissions. Now also CANZ is a permit seller. Market-clearing price established on the market of effective permits ($\frac{\partial c_i(l_i)}{\partial l_i}$) is 335,2 \$/tC. However, it is worth noting that marginal costs of reported emissions for each party ($\frac{\partial c_i(x_i)}{\partial x_i}$) at the equilibrium point differs, ranging from 295 \$/tC (EEFSU) to 322 \$/tC (OECDE). This reflects differentiated level of inventory uncertainty (Table 1). Total abatement costs has also increased dramatically achieving the sum of almost 180 000 MUS\$.

The situation evolves in the same direction as we decrease paremeter α. Finally, willing to require undershooting of the entire uncertainty belt Δ_i^{bc}, as defined in equation (4), we would have to accept the effective permit shadow price of 659,7 \$/tC and the sum of total abatement costs of 558 784 MUS\$. This was the reason to examine also adjusted Kyoto obligations according to (11).

	Effective emission permits	Reported emissions	Effective permits traded	A	B	Total costs
Units	MtC/year	MtC/year	MtC/year	\$/tC	\$/tC	MUS
Variable	l_i	x_i		$\frac{\partial c_i(x_i)}{\partial x_i}$	$\frac{\partial c_i(l_i)}{\partial l_i}$	$c_i(l_i)$
	$\alpha = 0,5$					
US	1561,6	1561,6	310,8	-142,5	-142,5	18433
OECDE	959,4	959,4	99,1	-142,5	-142,5	5602
Japan	321,1	321,1	63,5	-142,5	-142,5	2059
CANZ	248,4	248,4	32,9	-142,5	-142,5	4583
EEFSU	807,8	807,8	-506,3	-142,5	-142,5	6473
Total	3898,3	3898,3	0			37150
	$\alpha = 0,3$					
US	1178,8	1243,6	125,7	-317,8	-335,2	91645
OECDE	826,1	860,5	72,0	-321,8	-335,2	28562
Japan	268,9	286,1	57,7	-315,1	-335,2	10064
CANZ	159,7	173,5	-6,6	-308,4	-335,2	21461
EEFSU	625,4	710,6	-248,8	-295,0	-335,2	27732
Total	3058,9	3274,3	0			179464
	$\alpha = 0,1$					
US	834,4	931,2	-35,7	-489,9	-546,7	217773
OECDE	699,7	760,5	45,8	-503,0	-546,7	69778
Japan	222,2	252,5	53,4	-481,1	-546,7	23463
CANZ	88,6	105,5	-34,1	-459,3	-546,7	47591
EEFSU	481,7	633,8	-29,4	-415,5	-546,7	55023
Total	2326,6	2683,5	0			413628
	$\alpha = 0,0$					
US	677,5	778,7	-106,5	-573,9	-659,7	298904
OECDE	639,5	710,5	33,4	-593,7	-659,7	97214
Japan	200,9	236,3	51,8	-560,7	-659,7	31869
CANZ	59,6	74,6	-43,3	-527,7	-659,7	62843
EEFSU	423,0	604,3	64,6	-461,8	-659,7	67954
Total	2000,5	2404,4	0			558784

Table 2: Trading with effective permits according to the concept of undershooting for different levels of risk α - results at the equilibrium points; A - marginal cost of reported emission; B - marginal cost of effective permit

Trading with effective permits under adjustment. Adjustment of each party commitment obligation according to a reference uncertainty distribution has proven to be a practical solution. For $\alpha = 0,3; \alpha = 0,1; \alpha = 0$ the effective permit shadow price settles respectively: 170,6 \$/tC, 196,8 \$/tC and 208,5 \$/tC, the latter for the case of full treatment of uncertainty (Table 3). Smaller the risk α accepted, the more of inventory quality is covered in the system. For example, smaller the α, less of effective permits EEFSU is selling due to its highest inventory uncertainty. At the same time, OECDE with the lowest inventory uncertainty, has to buy less and less permits, turning from a net buyer into a net seller of effective permits.

Inclusion of uncertainty in the trading scheme bears some additional costs (total abatement costs in the equlibrium point) as compared to the standard system, even with the adjusted target level. It is inevitable under assumptions taken in the adjustment concept, as the abatement cost function is increasing and convex. However, under the presented adjusted commitment obligation those additional costs seem to be reasonable. They would increase from 37 150 MUS\$ to 55 850 MUS\$ i.e. by 50% when we consider full treatment of uncertainty. Other choice of the reference uncertainty R^M (see [5]) can possibly make this difference even less.

5 Concluding remarks and implications for further research

This exercise was to examine feasibility of incorporating uncertainty into the Kyoto protocol verification in the context of permit trading. The system of effective permits that reflect diversified inventory quality has been considered. The pure undershooting of the Kyoto target with uncertainty belt resulted in extremely high costs, that would be difficult to accept. Then a specific adjustment of the target with respect to a reference uncertainty gave reasonable results.

However, it is good to bear in mind also limitations of this analysis. An implicit assumption in the paper was that the inventory uncertainty is equally distributed over a certain interval. This is rather strong assumption and consideration of a stochastic setting is well justified. While adequate conditions for the undershooting and adjustment verification conditions has been already presented in [5], the definition of effective permits in a stochastic case still remains troublesome due to encountered nonlinearities. On the other hand, a stochastic case is the promising one. Not only it reflects more of reality, but also one may anticipate that abatement costs for the same risk α will be lower in a stochastic case with normal probability distributions than in an interval case. This is due to the effect of concentration of probability around the mean value. This may open the way towards furher decrease of abatement costs.

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	Effective emission permits	Reported emissions	Effective permits traded	A	B	Total costs
Units	MtC/year	MtC/year	MtC/year	\$/tC	\$/tC	MUS
Variable	l_i	x_i		$\frac{\partial c_i(x_i)}{\partial x_i}$	$\frac{\partial c_i(l_i)}{\partial l_i}$	$c_i(l_i)$
	$\alpha = 0,3$					
US	1447,5	1526,8	218,0	-161,7	-170,6	23726
OECDE	909,8	947,7	31,8	-163,7	-170,6	7395
Japan	298,4	317,5	51,6	-160,3	-170,6	2605
CANZ	222,5	241,9	28,7	-156,9	-170,6	5556
EEFSU	706,6	802,9	-330,1	-150,1	-170,6	7180
Total	3584,8	3836,8	0			46462
	$\alpha = 0,1$					
US	1334,3	1500,3	141,1	-176,3	-196,8	28210
OECDE	863,1	938,1	-28,3	-181,0	-196,8	9039
Japan	277,1	314,9	41,7	-173,2	-196,8	3039
CANZ	200,0	238,1	26,9	-165,3	-196,8	6165
EEFSU	610,5	803,3	-181,4	-149,5	-196,8	7128
Total	3285,0	3794,7	0			53581
	$\alpha = 0,0$					
US	1297,2	1491,0	108,9	-181,4	-208,5	29875
OECDE	841,0	934,5	-55,5	-187,2	-208,5	9716
Japan	266,9	314,0	37,4	-177,2	-208,5	3185
CANZ	189,9	237,4	27,0	-166,8	-208,5	6281
EEFSU	563,9	805,6	-117,8	-145,9	-208,5	6792
Total	3158,9	3782,5	0			55849

Table 3: Trading with effective permits according to adjusted Kyoto obligation for different levels of risk α - results at an equilibrium point; A - marginal cost of reported emission; B - marginal cost of effective permit

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